

ON A STOPPING TIME ALGORITHM OF THE $3n + 1$ FUNCTION

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ABSTRACT. The behaviour of a Collatz sequence is clearly related to the way in which the powers of 2 are distributed among the powers of 3 in a stopping time term formula. It is shown, that the seemingly chaotically distribution of the powers of 2 can be generated by an iterative algorithm (conjecture), which also allows an insight into the properties of the number $\log_2 3$. From Chapter 3 we are interested in only the odd starting numbers of the Collatz function. Chapter 1. – 3. of the this text are based on a text by Lynn E. Garner [1].

1. Introduction. The Collatz $3n + 1$ function is defined as a function $T: \mathbb{N} \rightarrow \mathbb{N}$ on the set of positive integers by

$$T(n) := \begin{cases} T_0(n) := \frac{n}{2} & \text{if } n \text{ is even,} \\ T_1(n) := \frac{3n+1}{2} & \text{if } n \text{ is odd.} \end{cases} \quad (1)$$

Let $T^0(s) = s$ and $T^k(s) = T(T^{k-1}(s))$ for $k \in \mathbb{N}$. The Collatz sequence for $s \in \mathbb{N}$ is

$$C(s) = \{T^k(s) \mid k = 0, 1, 2, \dots\}.$$

For example, $C(11) = \{11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2, 1, 2, 1, 2, 1, \dots\}$.

2. Stopping time. Collatz's conjecture is equivalent to the conjecture that for each $s \in \mathbb{N}$, $s > 1$, there exists $k \in \mathbb{N}$ such that $T^k(s) < s$. The least $k \in \mathbb{N}$ such that $T^k(s) < s$ is called the stopping time of s , which we will denote by $\sigma(s)$. It is not hard to verify that

$$\begin{aligned} \sigma(s) = 1 & \text{ if } s \equiv 0 \pmod{2}, \\ \sigma(s) = 2 & \text{ if } s \equiv 1 \pmod{4}, \\ \sigma(s) = 4 & \text{ if } s \equiv 3 \pmod{16}, \\ \sigma(s) = 5 & \text{ if } s \equiv 11, 23 \pmod{32}, \\ \sigma(s) = 7 & \text{ if } s \equiv 7, 15, 59 \pmod{128}, \\ \sigma(s) = 8 & \text{ if } s \equiv 39, 79, 95, 123, 175, 199, 219 \pmod{256}, \end{aligned}$$

and so forth.

T. D. Noe [2] has listed these numbers. J. C. Everett [3] proves that almost all $s \in \mathbb{N}$ have finite stopping time, and R. Terras [4] gives a probability distribution function for stopping times.

Let $\kappa(u) = \lceil u \cdot \log_2 3 \rceil$, so it is proofed, that for each $u \in \mathbb{N}$ there exists a set of r odd positive integers n_z , $z = 1, \dots, r$, defined by

$$\left. \begin{aligned} N_u &:= \{n_z \in \mathbb{N} \mid n_1 < \dots < n_r < 2^{\kappa(u)}\}, \\ \sigma(s) &= \kappa(u) \text{ if } s \equiv n_z \pmod{2^{\kappa(u)}}. \end{aligned} \right\} (2)$$

For simplifying, if z is not needed, we write n_z as n . Let $N^* = \{N_u \mid u \in \mathbb{N}\}$, then all elements of N^* are different.

3. A stopping time term formula for odd numbers. Let $C^k(s)$ consist of the first k terms of the Collatz sequence for odd s , and let u be the number of odd terms in $C^k(s)$. Further let $\alpha_i = k$, if and only if $T^k(s)$ is odd in $C^k(s)$, then for all $s \equiv n_z \pmod{2^{k(u)}}$ the next term in the Collatz sequence is

$$T^k(s) = \frac{3^u}{2^k} \cdot s + \sum_{i=1}^u \frac{3^{u-i} 2^{\alpha_i}}{2^k} < s, \quad k = \kappa(u). \quad (3)$$

In fact of that, if $s \equiv n_z \pmod{2^{k(u)}}$ the $(u + 1)$ th odd term in $C(s)$ is smaller than s .

For example, $C^5(11) = \{11, 17, 26, 13, 20\}$. $T^0 = 11$ is odd, so $\alpha_1 = 0$, $T^1 = 17$ is odd, so $\alpha_2 = 1$, $T^2 = 26$ is even, $T^3 = 13$ is odd, so $\alpha_3 = 3$, $T^4 = 20$ is even. With (3) we get

$$T^5(11) = \frac{3^3}{2^5} \cdot 11 + \frac{3^2 2^0 + 3^1 2^1 + 3^0 2^3}{2^5} = 10 < 11.$$

4. Binary matrices B_n . For each $u \in \mathbb{N}$ the first $u + 1$ terms in $C(s)$ represents sufficiently the stopping time of an odd number s , because all the other terms are even before an odd term $T^k(s) < s$ is reached. To simplify the distribution of the powers of 2 in $3^{u-i} 2^{\alpha_i}$, let “0” represents an even term and “1” represents an odd term in $C(s)$. For each $u \in \mathbb{N}$ let $B_n[u]$ be a binary $(1, u + 1)$ -matrix, defined by

$$B_n[u] = (b_{1,1} \quad \cdots \quad b_{1,u+1}) := \left(\begin{array}{l} b_{1,k+1} := 0 \quad \text{if } T^k(s) \text{ is even} \\ b_{1,k+1} := 1 \quad \text{if } T^k(s) \text{ is odd} \end{array} \right)_{k=0}^u. \quad (4)$$

Note that $b_{1,1} = b_{1,2} = 1$ for each $B_n[u]$.

For example, for $u = 3$ there are two numbers n_z , $n_1 = 11$ and $n_2 = 23$. Then $C(11) = \{11, 17, 26, 13, 20, 10, 5, \dots\}$ is represented by $B_{11}[3] = (1 \ 1 \ 0 \ 1)$, and $C(23) = \{23, 35, 53, 80, 40, 20, 10, 5, \dots\}$ is represented by $B_{23}[3] = (1 \ 1 \ 1 \ 0)$.

5. Binary matrices A_m . For $m \in \mathbb{N}$, with $3 \leq m \leq u - 1$, let A_m be an inverted anti-diagonal binary matrix of m rows and columns, defined by

$$A_m := \begin{pmatrix} a_{1,1} & \cdots & a_{1,m} \\ \vdots & \ddots & \vdots \\ a_{m,1} & \cdots & a_{m,m} \end{pmatrix}, \quad (5)$$

which all entries are “1”, except the diagonal from $a_{1,m}$ to $a_{m,1}$, which entries are “0”.

For example,

$$A_3 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

6. Sorting the matrices B_n . For each $u \in \mathbb{N}$ the numbers n_z seem to be distributed chaotically between 0 and $2^{K(u)}$. But if we sort the matrices B_n in a special way, then a system is seen in the arrangement of their elements, which can be transferred into an iterative algorithm.

To understand how the sorting-system is working, let us first take a look at the example for $u = 9$ on page 12.

For $u = 9$ there are 173 numbers n_z . With (4) we get 173 matrices B_n , which we can combine into 37 larger matrices S_m such a way, that we get a square-matrix A_m into each matrix S_m . The elements behind A_m of each S_m must be equal for each column.

The index “m” is the number of B_n for each S_m , and also be the number of rows and columns of A_m . The green square frame show A_m .

At least we sort the 37 matrices S_m into 9 sets G_t such a way, that A_m extends a column and a row from bottom to top. The last elements behind A_m must be equal in each set. The biggest matrix S_m of a set G_t contains “0”, “0 0” or no more elements behind A_m .

The index “t” of a set G_t is equal to the index “m” of the biggest matrix S_m . In fact of that a set G_t contains $t - 2$ matrices S_m .

The following chapters 7 and 8 give the mathematical correct definition of the matrices S_m and sets G_t .

7. Binary matrices S_m . For each $u > 3$ let $S_m[u]$ be a $(m, u + 1)$ -matrix, which rows consist of m *different* matrices $B_n[u]$, defined by

$$S_m[u] := \begin{pmatrix} s_{1,1} & \cdots & s_{1,u+1} \\ \vdots & \ddots & \vdots \\ s_{m,1} & \cdots & s_{m,u+1} \end{pmatrix} = \begin{pmatrix} B_n[u] \\ \vdots \\ B_n[u] \end{pmatrix},$$

which $B_n[u]$ sorted such a way, that

$$\forall S_m[u], \begin{pmatrix} s_{1,3} & \cdots & s_{1,m+2} \\ \vdots & \ddots & \vdots \\ s_{m,3} & \cdots & s_{m,m+2} \end{pmatrix} = A_m$$

and

$$s_{1,d}, \dots, s_{m,d} = s_{1,d}, \quad d = m + 3, \dots, u + 1.$$

} (6)

For example, for $u = 9$ we have among others $B_{1095}[9] = (1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1)$, $B_{9679}[9] = (1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1)$ and $B_{30715}[9] = (1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1)$. These three matrices can be sorted such a way, that we get by (6):

$$S_3[9] = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} B_{9679}[9] \\ B_{1095}[9] \\ B_{30715}[9] \end{pmatrix}.$$

Note A_3 in columns 3, 4, 5 and the same entries in each column followed.

8. Sets G_t of binary matrices S_m . For each $u > 5$ let $G_t[u]$ be a set of $t - 2$ matrices $S_m[u]$, defined by

$$\left. \begin{aligned} G_t[u] &:= \{S_m[u] \mid m = 3, \dots, t, 5 \leq t \leq u - 1\}, \\ \text{which } S_m[u] \text{ sorted such a way, that for } G_t[u], t = 5, \dots, u - 3, \text{ is} \\ (s_{1,m+5} \dots s_{1,u+1}) \text{ of } S_m[u] &= (s_{1,m+5} \dots s_{1,u+1}) \text{ of } S_{m+1}[u], \quad m = 3, \dots, t - 1 \\ \text{and for } G_{u-2}[u] \text{ and } G_{u-1}[u] \text{ is} \\ (s_{1,m+5} \dots s_{1,u+1}) \text{ of } S_m[u] &= (s_{1,m+5} \dots s_{1,u+1}) \text{ of } S_{m+1}[u], \quad m = 3, \dots, u - 4. \end{aligned} \right\} (7)$$

See Appendix A for a complete list of sorted matrices $B_n[u]$ for $u = 4, \dots, 9$.

9. Sorting theorem: The numbers of B_n , S_m and G_t for each u . Let $\gamma_k[u]$, $k = 1, \dots, u - 5$, be the number of $G_{u-k}[u]$, then for each $u > 5$ there exists $\gamma_{u-4}[u] \in \mathbb{N}$, defined by

$$\left. \begin{aligned} \gamma_{u-4}[u] &:= \sum_{k=1}^{u-5} \gamma_k[u], \\ \gamma_1[u] = 1 &< \gamma_2[u] < \gamma_3[u] < \dots < \gamma_{u-7}[u] < \gamma_{u-6}[u] = \gamma_{u-5}[u]. \end{aligned} \right\} (8)$$

Let $\gamma_{u-3}[u]$ be the number of all matrices $S_m[u]$ for each $u > 5$, defined by

$$\gamma_{u-3}[u] := \sum_{k=1}^{u-5} \gamma_k[u] \cdot (u - k - 2). \quad (9)$$

Let $\gamma_{u-2}[u]$ be the number of all matrices $B_n[u]$ for each $u > 5$, defined by

$$\gamma_{u-2}[u] := \sum_{k=1}^{u-5} \frac{\gamma_k[u] \cdot (u - k - 2)(u - k + 3)}{2}. \quad (10)$$

The following Table shows $\gamma_k[u]$ for $u = 6, \dots, 15$. The shaded elements show $\gamma_{u-4}[u]$, $\gamma_{u-3}[u]$, $\gamma_{u-2}[u]$.

u	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9	γ_{10}	γ_{11}	γ_{12}	γ_{13}
6	1	1	3	12									
7	1	1	2	7	30								
8	1	2	2	5	19	85							
9	1	2	3	3	9	37	173						
10	1	3	5	7	7	23	99	476					
11	1	3	6	9	12	12	43	194	961				
12	1	4	9	16	23	30	30	113	525	2652			
13	1	5	14	28	47	66	85	85	331	1570	8045		
14	1	5	15	34	62	99	136	173	173	698	3387	17637	
15	1	6	20	50	103	179	278	377	476	476	1966	9690	51033

For example, $\gamma_1[6] = 1$, $\gamma_2[6] = 1$, $\gamma_3[6] = 3$, $\gamma_4[6] = 12$.

Compare the entries of $u = 6, \dots, 9$ with the sorted matrices on page 10, 11 and 12. Appendix C shows a larger Table of $\gamma_k[u]$ for $u = 6, \dots, 30$.

10. Analyzing the entries of the Table. When analyzing $\gamma_k[u]$, $k \geq 6$, the following relationships are identified.

For $u = 11, \dots, 17$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5], \quad k \in \{10, 11, 12, 13\}\end{aligned}$$

For $u = 18, \dots, 29$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{10}[12], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_9[12], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_8[12], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{17-k}[12], \\ &\quad k \in \{10, \dots, 25\}\end{aligned}$$

and so forth. See Appendix D for a larger overview of these relationships.

11. The $\gamma_k[u]$ -Algorithm. We conjecture, that exists an iterative algorithm for each $u \geq 11$, which generates $\gamma_1[u], \dots, \gamma_{u-2}[u]$ by using $\gamma_{u-k}[u-w]$ for $k = 7, \dots, u-4$ and $w \geq 5$. Let K_1, K_2, K_3, K_4 be sets of positive integers defined by

$$K_1 := \{5\}, \quad K_2 := \{6, 7, 8\}, \quad K_3 := \{8, 9, 10\}, \quad K_4 := \{10, \dots, u-4\}.$$

Let $\gamma_{u-k}[u] = 0$ for $k \geq u$ and $m = \left\lfloor \frac{u-6}{12} \right\rfloor$. With $\gamma_k[6], \dots, \gamma_k[10]$ as starting values, the algorithm runs autonomously by using (8), (9), (10) and the following equations. Note that $\gamma_{u-5}[u] = \gamma_{u-6}[u]$.

$$\sum_{i=0}^0 \binom{1}{i} (-1)^i \gamma_{u-k-i}[u] = \gamma_{u-7}[u-5] - \sum_{n=1}^m \gamma_{u-12n-5}[u-12n] \cdot \gamma_{12n-2}[12n], \quad k \in K_1 \quad (11)$$

$$\sum_{i=0}^1 \binom{1}{i} (-1)^i \gamma_{u-k-i}[u] = \gamma_{u-8}[u-5] - \sum_{n=1}^m \gamma_{u-12n-5}[u-12n] \cdot \gamma_{12n-3}[12n], \quad k \in K_2 \quad (12)$$

$$\sum_{i=0}^2 \binom{2}{i} (-1)^i \gamma_{u-k-i}[u] = \gamma_{u-9}[u-5] - \sum_{n=1}^m \gamma_{u-12n-5}[u-12n] \cdot \gamma_{12n-4}[12n], \quad k \in K_3 \quad (13)$$

$$\sum_{i=0}^3 \binom{3}{i} (-1)^i \gamma_{u-k-i}[u] = \gamma_{u-k}[u-5] - \sum_{n=1}^m \gamma_{u-12n-5}[u-12n] \cdot \gamma_{12n+5-k}[12n], \quad k \in K_4 \quad (14)$$

Appendix B shows a Delphi/PASCAL Code for the $\gamma_k[u]$ -Algorithm. Appendix E shows a simple list of Appendix C for $u = 6, \dots, 77$.

For example, we generate the entries for $u = 16$. If we use (11) as initial term, we can get $\gamma_{11}[16], \dots, \gamma_1[16]$ by using $\gamma_9[11], \dots, \gamma_4[11]$ with (12), (13), (14) and $m = 0$.

$$\begin{aligned}\gamma_{11}[16] &= \gamma_9[11] = 961 \\ \gamma_{10}[16] &= \gamma_{11}[16] = 961\end{aligned}$$

$$\begin{aligned}\gamma_9[16] &= \gamma_{10}[16] - \gamma_8[11] = 961 - 194 = 767 \\ \gamma_8[16] &= \gamma_9[16] - \gamma_8[11] = 767 - 194 = 573 \\ \gamma_7[16] &= \gamma_8[16] - \gamma_8[11] = 573 - 194 = 379\end{aligned}$$

$$\begin{aligned}\gamma_6[16] &= 2\gamma_7[16] - \gamma_8[16] + \gamma_7[11] = 2 \cdot 379 - 573 + 43 = 228 \\ \gamma_5[16] &= 2\gamma_6[16] - \gamma_7[16] + \gamma_7[11] = 2 \cdot 228 - 379 + 43 = 120 \\ \gamma_4[16] &= 2\gamma_5[16] - \gamma_6[16] + \gamma_7[11] = 2 \cdot 120 - 228 + 43 = 55\end{aligned}$$

$$\begin{aligned}\gamma_3[16] &= 3\gamma_4[16] - 3\gamma_5[16] + \gamma_6[16] - \gamma_6[11] = 3 \cdot 55 - 3 \cdot 120 + 228 - 12 = 21 \\ \gamma_2[16] &= 3\gamma_3[16] - 3\gamma_4[16] + \gamma_5[16] - \gamma_5[11] = 3 \cdot 21 - 3 \cdot 55 + 120 - 12 = 6 \\ \gamma_1[16] &= 3\gamma_2[16] - 3\gamma_3[16] + \gamma_4[16] - \gamma_4[11] = 3 \cdot 6 - 3 \cdot 21 + 55 - 9 = 1\end{aligned}$$

and with (8), (9), (10) we get the values of $\gamma_{12}[16]$, $\gamma_{13}[16]$ and $\gamma_{14}[16]$.

$$\gamma_{12}[16] = \sum_{k=1}^{11} \gamma_k[16] = 1 + 6 + 21 + 55 + 120 + 228 + 379 + 573 + 767 + 961 + 961 = 4072$$

$$\begin{aligned}\gamma_{13}[16] &= \sum_{k=1}^{11} \gamma_k[16] \cdot (14 - k) = 1 \cdot 13 + 6 \cdot 12 + 21 \cdot 11 + 55 \cdot 10 + 120 \cdot 9 + 228 \cdot 8 + \\ & 379 \cdot 7 + 573 \cdot 6 + 767 \cdot 5 + 9679 \cdot 3 + 967 \cdot 2 = 20423\end{aligned}$$

$$\begin{aligned}\gamma_{14}[16] &= \sum_{k=1}^{11} \frac{\gamma_k[16] \cdot (14 - k)(19 - k)}{2} = \frac{1}{2} \cdot 1 \cdot 13 \cdot 18 + 6 \cdot 12 \cdot 17 + 21 \cdot 11 \cdot 16 + 55 \cdot 10 \cdot 15 + \\ & 120 \cdot 9 \cdot 14 + 228 \cdot 8 \cdot 13 + 379 \cdot 7 \cdot 12 + 573 \cdot 6 \cdot 11 + \\ & 767 \cdot 5 \cdot 10 + 9679 \cdot 3 \cdot 9 + 967 \cdot 2 \cdot 8 = 108950\end{aligned}$$

Now we have all entries for the row $u = 16$, which enable us to generate the entries for the row $u = 21$ and so forth.

u	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}
16	1	6	21	55	120	228	379	573	767	961	961	4072	20423	108950

12. How to get $B_n[u]$ for each u. Let $B^{m,k}[u]$ be the matrix in the m th row of each $S_m[u] \in G_{u-k}[u]$, defined by

$$\left. \begin{aligned} B^{m,k}[u] &:= (b_{1,1} = s_{m,1} \cdots b_{1,u+1} = s_{m,u+1}), \quad S_m[u] \in G_{u-k}[u], \\ m &= 3, \dots, u-1 \quad \text{and} \quad k = 1, \dots, u-5. \end{aligned} \right\} (15)$$

For example, for each $u > 3$, there exists $u-3$ matrices $B^{m,1}[u]$.

When analyzing the sorted matrices $B_n[u]$ of each $u > 3$, the following relationships between the entries of $B^{m,1}[u-3]$ and $B^{3,k}[u]$ are identified. For each

$$\left. \begin{aligned} & B^{3,k}[u], \quad k = 2, \dots, u-5, \\ \text{is } (b_{1,1} \cdots b_{1,3}) &= (1 \ 1 \ 0) \quad \text{and} \quad (b_{1,4} \cdots b_{1,u+1}) = B^{u-k-2,1}[u-3]. \end{aligned} \right\} (16)$$

$$\text{Further is} \quad B^{3,1}[u] = B^{3,2}[u]. \quad (17)$$

With other words, for $k = 2, \dots, u - 5$ the first three entries of each $B^{3,k}[u]$ are “1 1 0”, and the following $u - 2$ entries are the same as in $B^{u-k-2,1}[u - 3]$.

For example, for $u = 6$ there are twelve matrices B_n . After sorting them we get three matrices S_m , sorted into one set G_t . For simplifying, we dispense with the parentheses of matrices. The shaded elements show A_m . The matrices S_m of the set G_t are listed from bottom to top.

	n	$G_5[6]$
$S_5[6]$	575	1 1 1 1 1 0
	287	1 1 1 1 0 1
	367	1 1 1 1 0 1 1
	999	1 1 1 0 1 1 1
	923	1 1 0 1 1 1 1
$S_4[6]$	735	1 1 1 1 1 0 0
	815	1 1 1 1 0 1 0
	423	1 1 1 0 1 1 0
	347	1 1 0 1 1 1 0
$S_3[6]$	975	1 1 1 1 0 0 1
	583	1 1 1 0 1 0 1
	507	1 1 0 1 1 0 1

For $u = 6$ we have three matrices $B^{m,1}[6]$, marked by the purple frame.

$$B^{5,1}[6] = B_{923}[6] = (1\ 1\ 0\ 1\ 1\ 1\ 1)$$

$$B^{4,1}[6] = B_{347}[6] = (1\ 1\ 0\ 1\ 1\ 1\ 0)$$

$$B^{3,1}[6] = B_{507}[6] = (1\ 1\ 0\ 1\ 1\ 0\ 1)$$

These three matrices build the elements after “1 1 0” of the matrices $B^{3,1}[9]$, $B^{3,2}[9]$, $B^{3,3}[9]$ and $B^{3,4}[9]$.

Compare it with the sorted matrices of $u = 9$ on page 12 to see how (16) and (17) work. Note the equal elements.

13. Same matrices $B_n[u]$ with *different* n. As seen in Appendix A for $\gamma_k[u] > 1$, there exist $\gamma_k[u]$ matrices $B_n[u]$ with *different* n, which is due to the reduction to the first $u + 1$ elements. Further, for each $u > 6$, the matrices of $G_{u-2}[u]$ exist $\gamma_2[u] + 1$ times, because $G_{u-2}[u] = \{G_{u-1}[u] \mid S_{u-1}[u] \notin G_{u-1}[u]\}$. In Chapter 15 we will more accurately respond to this special feature.

14. The $B_n[u]$ -Algorithm. We conjecture, that exists an algorithm for each $u \geq 9$, which generates each $B_n[u]$ by using $B^{m,1}[u - 3]$, $m = 3, \dots, u - 1$.

First we build $S_3[u] \in G_{u-k}[u]$, $k = 2, \dots, u - 5$, by using (16) and (6) as shown by (18).

$$\left. \begin{aligned}
 S_3[u] \in G_{u-k}[u] &= \left(\begin{array}{cccccc} 1 & 1 & 1 & 1 & 0 & 0 & s_{1,7} & \cdots & s_{1,u+1} \\ 1 & 1 & 1 & 0 & 1 & 0 & s_{2,7} & \cdots & s_{2,u+1} \\ 1 & 1 & 0 & 1 & 1 & 0 & s_{3,7} & \cdots & s_{3,u+1} \end{array} \right), \quad k = 2, \dots, u - 5 \\
 \text{with} & \quad (s_{3,4} \ \cdots \ s_{3,u+1}) = B^{u-k-2,1}[u - 3] \\
 \text{and} & \quad (s_{3,6} \ \cdots \ s_{3,u+1}) = (s_{2,6} \ \cdots \ s_{2,u+1}) = (s_{1,6} \ \cdots \ s_{1,u+1}).
 \end{aligned} \right\} (18)$$

Then we build $S_4, \dots, S_{u-k}[u] \in G_{u-k}[u]$, $k = 2, \dots, u - 5$, by using (7) as shown by (19).

$$\left. \begin{aligned} S_m[u] \in G_{u-k}[u] &= \begin{pmatrix} 1 & 1 & s_{1,3} & \cdots & s_{1,m+2} & 0 & s_{1,m+4} & \cdots & s_{1,u+1} \\ 1 & 1 & \vdots & \ddots & \vdots & 0 & \vdots & \ddots & \vdots \\ 1 & 1 & s_{m,3} & \cdots & s_{m,m+2} & 0 & s_{m,m+4} & \cdots & s_{m,u+1} \end{pmatrix}, \\ \begin{pmatrix} s_{1,3} & \cdots & s_{1,m+2} \\ \vdots & \ddots & \vdots \\ s_{m,3} & \cdots & s_{m,m+2} \end{pmatrix} &= A_m, \quad m = 4, \dots, u - k \quad \text{and} \quad k = 2, \dots, u - 5, \end{aligned} \right\} (19)$$

until the biggest matrix $S_{u-k}[u] \in G_{u-k}[u]$ as shown by (20) is reached.

$$S_{u-k}[u] \in G_{u-k}[u] = \begin{pmatrix} 1 & 1 & s_{1,3} & \cdots & s_{1,m+2} & 0 & 0 & s_{1,m+5} & \cdots & s_{1,u+1} \\ 1 & 1 & \vdots & \ddots & \vdots & 0 & 0 & \vdots & \ddots & \vdots \\ 1 & 1 & s_{m,3} & \cdots & s_{m,m+2} & 0 & 0 & s_{m,m+5} & \cdots & s_{m,u+1} \end{pmatrix}. \quad (20)$$

The biggest set $G_{u-1}[u]$ is build as shown by (21).

$$G_{u-1}[u] = G_{u-2}[u], \quad S_{u-1}[u] \in G_{u-1}[u] = \begin{pmatrix} 1 & 1 & s_{1,3} & \cdots & s_{1,m+2} \\ 1 & 1 & \vdots & \ddots & \vdots \\ 1 & 1 & s_{m,3} & \cdots & s_{m,m+2} \end{pmatrix}. \quad (21)$$

At least, the number of sets $G_{u-k}[u]$ we get by (8) and the $\gamma_k[u]$ -Algorithm.

15. Summary. With both algorithms we are in possession of tools to generate each $B_n[u]$ for $u \geq 11$. With the starting values of $\gamma_k[6], \dots, \gamma_k[10]$ the $\gamma_k[u]$ -Algorithm yields the numbers of matrices $B_n[u]$ and $S_m[u]$ and sets $G_i[u]$. The $B_n[u]$ -Algorithm then yields the exact nature of matrices $B_n[u]$. (*reduced to the first $u + 1$ elements*)

But how to get each $n_z \in N_u$?

As mentioned in chapter 13, there exist same matrices $B_n[u]$ with *different* n_z for each $u \geq 6$. Remember a matrix $B_n[u]$ is a simplification for the distribution of the powers of 2 in $3^{u-1}2^{\alpha_i}$ in (3). Therefore, if (3) is interpreting as a Diophantine equation as shown by (22), then for each $B_n[u]$ the equation has $\gamma_k[u]$ different solutions $x = n_z$.

$$2y + 1 = \frac{3^u}{2^k} \cdot x + \sum_{i=1}^u \frac{3^{u-i}2^{\alpha_i}}{2^k}, \quad k = \kappa(u) \quad (22)$$

Finally, the algorithms also allow an insight into the largely inscrutable properties of the number $\log_2 3 = 1,5849625\dots$, because they “replace” the expression $\kappa(u) = \lceil u \cdot \log_2 3 \rceil$ in (2) and (3) by elementary arithmetic. Most other work on the Collatz problem can not do without this mysterious number.

For example, the integer sequence by Finch [5] represents the numbers of $n_z \in \mathbb{N}_u$ (values of $\gamma_{u-2}[u]$), and the integer sequence by Noe [2] represents \mathbb{N}^* . Both sequences are generated by using $\log_2 3$ or the distinction $\log 3 < \log 2$. There is a Mathematica code for each sequence under the appropriate web-link.

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Appendix A. Complete list of sorted matrices $B_n[u]$ for $u = 4, \dots, 9$. For simplifying, we dispense with the parentheses of matrices. The matrices $S_m[u]$ of each set $G_t[u]$ are listed in columns from bottom to top. The shaded elements show A_m .

				u = 6				
						n	$G_5[6]$	
				u = 5		575	1 1 1 1 1 0	
						287	1 1 1 1 1 0 1	
						$S_5[6]$	367	1 1 1 1 0 1 1
						999	1 1 1 0 1 1 1	
						923	1 1 0 1 1 1 1	
						735	1 1 1 1 1 0 0	
						$S_4[6]$	815	1 1 1 1 0 1 0
						423	1 1 1 0 1 1 0	
						347	1 1 0 1 1 1 0	
						975	1 1 1 1 0 0 1	
						$S_3[6]$	583	1 1 1 0 1 0 1
						507	1 1 0 1 1 0 1	
						95	1 1 1 1 1 0	
						175	1 1 1 1 0 1	
						39	1 1 1 0 1 1	
						219	1 1 0 1 1 1	
						79	1 1 1 1 0 0	
						$S_3[5]$	199	1 1 1 0 1 0
						123	1 1 0 1 1 0	
						15	1 1 1 1 0	
						7	1 1 1 0 1	
						59	1 1 0 1 1	

u = 7								
				n	$G_6[7]$	n	$G_5[7]$	
				383	1 1 1 1 1 1 0			
				2239	1 1 1 1 1 1 0 1			
				$S_6[7]$	2975	1 1 1 1 1 0 1 1		
				2031	1 1 1 1 0 1 1 1			
				615	1 1 1 0 1 1 1 1			
				2587	1 1 0 1 1 1 1 1			
				1885	1 1 1 1 1 1 0 0	1087	1 1 1 1 1 1 0 0	
				1823	1 1 1 1 1 0 1 0	2591	1 1 1 1 1 0 1 0	
				$S_5[7]$	1647	1 1 1 1 0 1 1 0	879	1 1 1 1 0 1 1 0
				3559	1 1 1 0 1 1 1 0	231	1 1 1 0 1 1 1 0	
				2203	1 1 0 1 1 1 1 0	1435	1 1 0 1 1 1 1 0	
				4063	1 1 1 1 1 0 0 1	3295	1 1 1 1 1 0 0 1	
				$S_4[7]$	3119	1 1 1 1 0 1 0 1	2351	1 1 1 1 0 1 0 1
				1703	1 1 1 0 1 1 0 1	935	1 1 1 0 1 1 0 1	
				3675	1 1 0 1 1 1 0 1	2907	1 1 0 1 1 1 0 1	
				1231	1 1 1 1 0 0 1 1	463	1 1 1 1 0 0 1 1	
				$S_3[7]$	3911	1 1 1 0 1 0 1 1	3143	1 1 1 0 1 0 1 1
				1787	1 1 0 1 1 0 1 1	1019	1 1 0 1 1 0 1 1	
				59	1 1 0 1 1	59	1 1 0 1 1	

The matrix below the purple lines show $B^{3,1}[4]$. Note the equal elements.

For $u = 9$ the matrices $S_m[u]$ look something different. The green square frame show A_m . The matrix below a purple line show $B^{m,1}[u - 3]$. Note the equal elements. The numbers n of equal sets $G_i[u]$ are not in special order.

$u = 9$

	n	$G_8[9]$	n	$G_7[9]$	n	$G_7[9]$		
$S_8[9]$		<pre> 5631 1 1 1 1 1 1 1 1 0 19199 1 1 1 1 1 1 1 0 1 6783 1 1 1 1 1 1 0 1 1 20927 1 1 1 1 1 0 1 1 1 9375 1 1 1 1 1 0 1 1 1 8431 1 1 1 0 1 1 1 1 1 23399 1 1 1 0 1 1 1 1 1 29467 1 1 0 1 1 1 1 1 1 </pre>						
$S_7[9]$		<pre> 12543 1 1 1 1 1 1 1 0 0 22655 1 1 1 1 1 1 0 1 0 4031 1 1 1 1 1 0 1 1 0 3743 1 1 1 1 0 1 1 1 0 2799 1 1 1 1 0 1 1 1 0 16743 1 1 1 0 1 1 1 1 0 23835 1 1 0 1 1 1 1 1 0 </pre>		<pre> 13567 1 1 1 1 1 1 1 0 0 1151 1 1 1 1 1 1 0 1 0 14271 1 1 1 1 1 0 1 1 0 25247 1 1 1 1 0 1 1 1 0 24303 1 1 1 1 0 1 1 1 0 6503 1 1 0 1 1 1 1 1 0 12571 1 1 0 1 1 1 1 1 0 </pre>		<pre> 2303 1 1 1 1 1 1 1 0 0 127 1 1 1 1 1 1 0 1 0 15295 1 1 1 1 1 0 1 1 0 2719 1 1 1 1 1 0 1 1 0 1775 1 1 1 0 1 1 1 1 0 17767 1 1 0 1 1 1 1 1 0 22811 1 1 0 1 1 1 1 1 0 </pre>	$S_7[9]$	
$S_6[9]$		<pre> 8063 1 1 1 1 1 1 0 0 1 22207 1 1 1 1 1 0 1 0 1 10655 1 1 1 1 0 1 1 0 1 10735 1 1 1 1 0 1 1 1 0 14439 1 1 1 0 1 1 1 1 0 31771 1 1 0 1 1 1 1 1 0 </pre>		<pre> 9087 1 1 1 1 1 1 0 0 1 23231 1 1 1 1 1 0 1 0 1 415 1 1 1 1 0 1 1 0 1 9711 1 1 1 0 1 1 1 1 0 24679 1 1 1 0 1 1 1 1 0 20507 1 1 0 1 1 1 1 1 0 </pre>		<pre> 30591 1 1 1 1 1 1 0 0 1 11967 1 1 1 1 1 0 1 0 1 11679 1 1 1 1 0 1 1 0 1 32239 1 1 1 1 0 1 1 1 0 25703 1 1 1 0 1 1 1 1 0 30747 1 1 0 1 1 1 1 1 0 </pre>	$S_6[9]$	
$S_5[9]$		<pre> 17727 1 1 1 1 1 0 0 0 1 28703 1 1 1 1 1 0 0 1 1 6255 1 1 1 0 1 1 0 1 1 9959 1 1 1 0 1 1 1 0 1 16027 1 1 0 1 1 1 1 0 1 </pre>		<pre> 7487 1 1 1 1 1 0 0 1 1 7199 1 1 1 1 0 1 0 1 1 27759 1 1 1 0 1 1 0 1 1 20199 1 1 1 0 1 1 0 1 1 26267 1 1 0 1 1 1 1 0 1 </pre>		<pre> 18751 1 1 1 1 1 1 0 0 1 4175 1 1 1 1 0 1 1 0 1 5231 1 1 1 0 1 1 0 1 1 21223 1 1 1 0 1 1 1 0 1 27291 1 1 0 1 1 1 1 0 1 </pre>	$S_5[9]$	
$S_4[9]$		<pre> 16863 1 1 1 1 1 0 0 1 1 14895 1 1 1 1 0 1 0 1 1 30887 1 1 1 0 1 1 0 1 1 25691 1 1 0 1 1 1 0 1 1 </pre>		<pre> 15839 1 1 1 1 1 0 0 1 1 15919 1 1 1 1 0 1 0 1 1 29863 1 1 1 0 1 1 0 1 1 3163 1 1 0 1 1 1 0 1 1 </pre>		<pre> 5599 1 1 1 1 1 0 0 1 1 4655 1 1 1 1 0 1 0 1 1 19623 1 1 1 0 1 1 0 1 1 4187 1 1 0 1 1 1 0 1 1 </pre>	$S_4[9]$	
$S_3[9]$		<pre> 14031 1 1 1 1 0 0 0 1 1 27975 1 1 1 0 1 0 0 1 1 23603 1 1 1 0 1 0 0 1 1 </pre>		<pre> 13007 1 1 1 1 0 0 0 1 1 28999 1 1 1 0 1 0 0 1 1 1275 1 1 0 1 1 0 0 1 1 </pre>		<pre> 2767 1 1 1 1 0 0 0 1 1 17735 1 1 1 0 1 0 0 1 1 2299 1 1 0 1 1 0 0 1 1 </pre>	$S_3[9]$	
	923	1 1 0 1 1 1 1 1	923	1 1 0 1 1 1 1 1	923	1 1 0 1 1 1 1 1		

	n	$G_6[9]$	n	$G_6[9]$	n	$G_6[9]$	
$S_6[9]$		<pre> 22911 1 1 1 1 1 1 0 0 0 27839 1 1 1 1 1 1 0 1 0 26527 1 1 1 1 0 1 1 0 0 24559 1 1 1 1 0 1 1 1 0 30311 1 1 1 0 1 1 1 1 0 12827 1 1 0 1 1 1 1 1 0 </pre>		<pre> 23935 1 1 1 1 1 1 1 0 0 531 1 1 1 1 1 1 0 1 0 16287 1 1 1 1 0 1 1 1 0 15343 1 1 1 1 0 1 1 1 0 6759 1 1 1 0 1 1 1 1 0 3611 1 1 0 1 1 1 1 1 0 </pre>		<pre> 13695 1 1 1 1 1 1 1 0 0 4287 1 1 1 1 1 1 0 1 0 25503 1 1 1 1 1 0 1 1 0 25566 1 1 1 1 0 1 1 1 0 7783 1 1 1 0 1 1 1 1 0 13851 1 1 0 1 1 1 1 1 0 </pre>	$S_6[9]$
$S_5[9]$		<pre> 23359 1 1 1 1 1 1 0 0 1 21023 1 1 1 1 1 0 1 0 1 20079 1 1 1 1 0 1 1 0 1 3303 1 1 1 1 0 1 1 1 0 9371 1 1 0 1 1 1 1 1 0 </pre>		<pre> 831 1 1 1 1 1 1 0 0 1 11807 1 1 1 1 1 0 1 0 1 16287 1 1 1 1 0 1 1 1 0 25831 1 1 1 0 1 1 1 1 0 31899 1 1 0 1 1 1 1 1 0 </pre>		<pre> 32575 1 1 1 1 1 1 0 0 1 22047 1 1 1 1 1 0 1 0 1 21103 1 1 1 1 0 1 1 0 1 2279 1 1 1 0 1 1 1 1 0 8347 1 1 0 1 1 1 1 1 0 </pre>	$S_5[9]$
$S_4[9]$		<pre> 21471 1 1 1 1 1 0 0 1 1 20527 1 1 1 1 0 1 0 1 1 11943 1 1 1 0 1 1 0 1 1 18011 1 1 0 1 1 1 0 1 1 </pre>		<pre> 30687 1 1 1 1 1 0 0 1 1 30767 1 1 1 1 0 1 0 1 1 2727 1 1 1 0 1 1 0 1 1 19035 1 1 0 1 1 1 0 1 1 </pre>		<pre> 31711 1 1 1 1 1 0 0 1 1 29743 1 1 1 1 0 1 0 1 1 12967 1 1 1 0 1 1 0 1 1 8795 1 1 0 1 1 1 0 1 1 </pre>	$S_4[9]$
$S_3[9]$		<pre> 27855 1 1 1 1 0 0 0 1 1 10055 1 1 1 0 1 0 0 1 1 17147 1 1 0 1 1 0 0 1 1 </pre>		<pre> 18639 1 1 1 1 0 0 0 1 1 839 1 1 1 0 1 0 0 1 1 6907 1 1 0 1 1 0 0 1 1 </pre>		<pre> 28879 1 1 1 1 0 0 0 1 1 11079 1 1 1 0 1 0 0 1 1 16123 1 1 0 1 1 0 0 1 1 </pre>	$S_3[9]$
	347	1 1 0 1 1 1 1 0	347	1 1 0 1 1 1 1 0	347	1 1 0 1 1 1 1 0	

	n	$G_5[9]$	n	$G_5[9]$	n	$G_5[9]$	
$S_5[9]$		<pre> 14399 1 1 1 1 1 1 0 0 0 1 12063 1 1 1 1 1 0 1 0 0 1 11119 1 1 1 1 0 1 1 0 0 1 16871 1 1 1 0 1 1 1 0 0 1 22939 1 1 0 1 1 1 1 1 0 0 1 </pre>		<pre> 24639 1 1 1 1 1 1 0 0 0 1 2847 1 1 1 1 1 0 1 0 0 1 12143 1 1 1 1 0 1 1 0 0 1 27111 1 1 1 0 1 1 1 1 0 1 411 1 1 0 1 1 1 1 1 0 0 1 </pre>		<pre> 23615 1 1 1 1 1 1 0 0 0 1 13087 1 1 1 1 1 0 0 1 0 1 1903 1 1 1 1 0 1 1 0 0 1 26087 1 1 1 0 1 1 1 1 0 0 1 32155 1 1 0 1 1 1 1 1 0 0 1 </pre>	$S_5[9]$
$S_4[9]$		<pre> 21727 1 1 1 1 1 0 0 0 1 0 1 11567 1 1 1 1 0 1 0 1 0 1 4007 1 1 1 0 1 1 0 1 0 1 9051 1 1 0 1 1 1 0 1 0 1 </pre>		<pre> 12511 1 1 1 1 1 0 0 1 0 1 21607 1 1 1 1 0 1 0 1 0 1 26535 1 1 1 0 1 1 0 1 0 1 10075 1 1 0 1 1 1 0 1 0 1 </pre>		<pre> 22751 1 1 1 1 1 0 0 1 0 1 20783 1 1 1 1 0 1 0 1 0 1 2983 1 1 1 0 1 1 0 1 0 1 32603 1 1 0 1 1 1 0 1 0 1 </pre>	$S_4[9]$
$S_3[9]$		<pre> 9679 1 1 1 1 0 0 0 1 1 0 1 1095 1 1 1 0 1 0 0 1 1 0 1 30715 1 1 0 1 1 0 0 1 1 0 1 </pre>		<pre> 19919 1 1 1 1 0 0 0 1 1 0 1 24647 1 1 1 0 1 0 0 1 1 0 1 7163 1 1 0 1 1 0 0 1 1 0 1 </pre>		<pre> 18895 1 1 1 1 0 0 0 1 1 0 1 2119 1 1 1 0 1 0 0 1 1 0 1 8187 1 1 0 1 1 0 0 1 1 0 1 </pre>	$S_3[9]$
	507	1 1 0 1 1 1 0 1	507	1 1 0 1 1 1 0 1	507	1 1 0 1 1 1 0 1	

Appendix B. A Delphi/PASCAL Code for the $\gamma_k[u]$ -Algorithm. Of course it is possible to program the algorithm even more compact. In any case, the PASCAL code is a simple way to understand how the algorithm works and also it is a good basis for translation into other languages.

```

const max = 100; // maximum limit of u

var
  u,k,i,b: integer;
  a,m: extended;
  g: array[-max..max, - max..max] of extended;

begin
  g[-max..max, - max..max]:=0 // set all arrays = 0 (Pseudocode)

  // u = 6..10 starting values
  g[1,6]:=1; g[2,6]:=1; g[3,6]:=3; g[4,6]:=12;
  g[1,7]:=1; g[2,7]:=1; g[3,7]:=2; g[4,7]:=7; g[5,7]:=30;
  g[1,8]:=1; g[2,8]:=2; g[3,8]:=2; g[4,8]:=5; g[5,8]:=19; g[6,8]:=85;
  g[1,9]:=1; g[2,9]:=2; g[3,9]:=3; g[4,9]:=3; g[5,9]:=9; g[6,9]:=37; g[7,9]:=173;
  g[1,10]:=1; g[2,10]:=3; g[3,10]:=5; g[4,10]:=7; g[5,10]:=7; g[6,10]:=23; g[7,10]:=99;
  g[8,10]:=476;

  u:=11;
  repeat
    m:=floor((u-6)/12);

    k:=5; a:=0; b:=1;
    repeat
      a:= a + g[u-12*b-5,u-12*b]*g[12*b-2,12*b];
      b:=b+1;
    until b>m;
    g[u-k,u]:= g[u-7,u-5] - a;
    g[u-k-1,u]:= g[u-5,u];

    k:=6; a:=0; b:=1;
    repeat
      a:= a + g[u-12*b-5,u-12*b]*g[12*b-3,12*b];
      b:=b+1;
    until b>m;
    repeat
      g[u-k-1,u]:= g[u-k,u] - g[u-8,u-5] + a;
      k:=k+1;
    until k>8;

    k:=8; a:=0; b:=1;
    repeat
      a:= a + g[u-12*b-5,u-12*b]*g[12*b-4,12*b];
      b:=b+1;
    until b>m;
    repeat
      g[u-k-2,u]:= -g[u-k,u] + 2*g[u-k-1,u] + g[u-9,u-5] - a;
      k:=k+1;
    until k>10;

    k:=10;
    repeat
      a:=0; b:=1;
      repeat
        a:= a + g[u-12*b-5,u-12*b]*g[12*b+5-k,12*b];
        b:=b+1;
      until b>m;
      g[u-k-3,u]:= g[u-k,u] - 3*g[u-k-1,u] + 3*g[u-k-2,u] - g[u-k,u-5] + a;
      k:=k+1;
    until k>12*(m+1)+1;

    i:=1; g[u-4,u]:=0; g[u-3,u]:=0; g[u-2,u]:=0;
    repeat
      g[u-4,u]:=g[u-4,u]+g[i,u];
      g[u-3,u]:=g[u-3,u]+g[i,u]*(u-i-2);
      g[u-2,u]:=g[u-2,u]+g[i,u]*(u-i-2)*(u-i+3)/2;
      i:=i+1;
    until i>u-5;

    u:=u+1;
  until u>max;

  // output of values as table (Pseudocode)
  u:=1;
  repeat
    list[ u, g[1,u], g[2,u], g[3,u], ... , g[max,u] ];
    u:=u+1;
  until u>max;
end;

```

Appendix C. Table of $\gamma_k[u]$ for $u = 6, \dots, 30$. Please use the zoom function for better reading.

u	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}	γ_{18}	γ_{19}	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}	γ_{28}	
6	1	1	3	12																									
7	1	1	2	7	30																								
8	1	2	2	5	19	85																							
9	1	2	3	3	9	37	173																						
10	1	3	5	7	7	23	99	476																					
11	1	3	6	9	12	12	43	194	961																				
12	1	4	9	16	23	30	30	113	525	2652																			
13	1	5	14	28	47	66	85	331	1570	8045																			
14	1	5	15	34	62	99	136	173	698	3387	17637																		
15	1	6	20	50	103	179	278	377	476	1966	9690	51033																	
16	1	6	21	55	120	228	379	573	767	961	4072	20423	108950																
17	1	7	27	77	180	366	665	1077	1602	2127	2652	11433	58040	312455															
18	1	7	28	83	203	431	822	1431	2258	3303	4348	5393	5393	23701	121977	663535													
19	1	8	35	112	292	658	1323	2430	4122	6399	9261	12123	14985	14985	66734	346769	1900470												
20	1	9	44	154	434	1045	2233	4329	7749	12909	19809	28449	37089	45729	45729	205712	1076930	5936673											
21	1	9	45	164	483	1218	2721	5517	10304	17953	29335	44450	63298	82146	100994	100994	459632	2427209	13472296										
22	1	10	54	210	658	1757	4144	8833	17314	31553	53992	87073	130796	185161	239526	293891	293891	1348864	7168714	39993895									
23	1	10	55	219	705	1944	4746	10494	21338	40389	71719	120361	191348	284680	400357	516034	631711	631711	2927822	15676836	87986917								
24	1	11	65	275	933	2697	6890	15912	33765	66578	123132	214860	355847	560178	827853	1158872	1489891	1820910	1820910	8499580	45760565	257978502							
25	1	12	77	350	1265	3867	10389	25160	55844	115010	221702	403009	694065	1138049	1778140	2614338	3646643	4678948	5711253	5711253	26809375	144973012	820236724						
26	1	12	78	363	1350	4257	11805	29511	67679	144089	287384	540457	963838	1635694	2651829	4108047	6004348	8340732	10677116	13013500	13013500	61495590	334283132	1899474678					
27	1	13	90	442	1727	5702	16511	42987	102444	226435	468476	913736	1688727	2970994	4989115	8022701	12351363	17975101	24893915	31812729	38731543	38731543	183946295	1003880965	5723030586				
28	1	13	91	454	1807	6094	18061	48202	117855	267409	568583	1140777	2171495	3936768	6821577	11319853	18034477	27568330	39921412	55093723	70266034	85438345	85438345	408179706	2238139010	12809477536			
29	1	14	104	546	2273	7982	24563	67928	171842	402883	884183	1830602	3595335	6727952	12032438	20625233	33935232	53703785	81672242	117840603	162208868	206577133	250945398	250945398	1204202538	6626333161	38036848410		
30	1	14	105	559	2366	8460	26533	74855	193358	463287	1039766	2202672	4431210	8502189	15611999	27498588	46563439	75871570	119151534	180131884	258812620	355193742	451574864	547955986	547955986	2643267587	14606430265	84141805077	

Appendix D. Relationships of $\gamma_k[u]$ for $u = 11, \dots, 53$.

For $u = 11, \dots, 17$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5], \quad k \in \{10, \dots, u-4\}\end{aligned}$$

For $u = 18, \dots, 29$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{10}[12], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_9[12], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_8[12], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{17-k}[12], \quad k \in \{10, \dots, u-4\}\end{aligned}$$

For $u = 30, \dots, 41$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{10}[12] - \gamma_{u-29}[u-24] \cdot \gamma_{22}[24], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_9[12] - \gamma_{u-29}[u-24] \cdot \gamma_{21}[24], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_8[12] - \gamma_{u-29}[u-24] \cdot \gamma_{20}[24], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{17-k}[12] - \gamma_{u-29}[u-24] \cdot \gamma_{29-k}[24], \quad k \in \{10, \dots, u-4\}\end{aligned}$$

For $u = 42, \dots, 53$

$$\begin{aligned}\gamma_{u-k}[u] &= \gamma_{u-7}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{10}[12] - \gamma_{u-29}[u-24] \cdot \gamma_{22}[24] - \gamma_{u-41}[u-36] \cdot \gamma_{34}[36], \quad k \in \{5\} \\ \gamma_{u-k}[u] - \gamma_{u-k-1}[u] &= \gamma_{u-8}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_9[12] - \gamma_{u-29}[u-24] \cdot \gamma_{21}[24] - \gamma_{u-41}[u-36] \cdot \gamma_{33}[36], \quad k \in \{6, 7, 8\} \\ \gamma_{u-k}[u] - 2\gamma_{u-k-1}[u] + \gamma_{u-k-2}[u] &= \gamma_{u-9}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_8[12] - \gamma_{u-29}[u-24] \cdot \gamma_{20}[24] - \gamma_{u-41}[u-36] \cdot \gamma_{32}[36], \quad k \in \{8, 9, 10\} \\ \gamma_{u-k}[u] - 3\gamma_{u-k-1}[u] + 3\gamma_{u-k-2}[u] - \gamma_{u-k-3}[u] &= \gamma_{u-k}[u-5] - \gamma_{u-17}[u-12] \cdot \gamma_{17-k}[12] - \gamma_{u-29}[u-24] \cdot \gamma_{29-k}[24] - \gamma_{u-41}[u-36] \cdot \gamma_{41-k}[36], \quad k \in \{10, \dots, u-4\}\end{aligned}$$

and so forth, but slightly modified.

Appendix E. Simple list of Appendix C for $u = 6, \dots, 77$.

6
1 1 3 12

7
1 1 2 7 30

8
1 2 2 5 19 85

9
1 2 3 3 9 37 173

10
1 3 5 7 7 23 99 476

11
1 3 6 9 12 12 43 194 961

12
1 4 9 16 23 30 30 113 525 2652

13
1 5 14 28 47 66 85 85 331 1570 8045

14
1 5 15 34 62 99 136 173 173 698 3387 17637

15
1 6 20 50 103 179 278 377 476 476 1966 9690 51033

16
1 6 21 55 120 228 379 573 767 961 961 4072 20423 108950

17
1 7 27 77 180 366 665 1077 1602 2127 2652 2652 11433 58040 312455

18
1 7 28 83 203 431 822 1431 2258 3303 4348 5393 5393 23701 121977 663535

19
1 8 35 112 292 658 1323 2430 4122 6399 9261 12123 14985 14985 66734 346769 1900470

20
1 9 44 154 434 1045 2233 4329 7749 12909 19809 28449 37089 45729 45729 205712 1076930 5936673

21
1 9 45 164 483 1218 2721 5517 10304 17953 29335 44450 63298 82146 100994 100994 459632 2427209 13472296

22
1 10 54 210 658 1757 4144 8833 17314 31553 53992 87073 130796 185161 239526 293891 293891 1348864 7168714 39993895

23
1 10 55 219 705 1944 4746 10494 21338 40389 71719 120361 191348 284680 400357 516034 631711 631711 2927822 15676836 87986917

24

1 11 65 275 933 2697 6890 15912 33765 66578 123132 214860 355847 560178 827853 1158872 1489891 1820910 1820910 8499580 45760565 257978502

25

1 12 77 350 1265 3867 10389 25160 55844 115010 221702 403009 694065 1138049 1778140 2614338 3646643 4678948 5711253 5711253 26809375 144973012 820236724

26

1 12 78 363 1350 4257 11805 29511 67679 144089 287384 540457 963838 1635694 2651829 4108047 6004348 8340732 10677116 13013500 13013500 61495590 334283132 1899474678

27

1 13 90 442 1727 5702 16511 42987 102444 226435 468476 913736 1688727 2970994 4989115 8022701 12351363 17975101 24893915 31812729 38731543 38731543 183946295 1003880965 5723030586

28

1 13 91 454 1807 6094 18061 48202 117855 267409 568583 1140777 2171495 3936768 6821577 11319853 18034477 27568330 39921412 55093723 70266034 85438345 85438345 408179706 2238139010 12809477536

29

1 14 104 546 2273 7982 24563 67928 171842 402883 884183 1830602 3595335 6727952 12032438 20625233 33935232 53703785 81672242 117840603 162208868 206577133 250945398 250945398 1204202538 6626333161 38036848410

30

1 14 105 559 2366 8460 26533 74855 193358 463287 1039766 2202672 4431210 8502189 15611999 27498588 46563439 75871570 119151534 180131884 258812620 355193742 451574864 547955986 547955986 2643267587 14606430265 84141805077

31

1 15 119 665 2938 10920 35483 103441 275583 680086 1570569 3420911 7071818 13944124 26318826 47683853 83080835 139451872 225639534 352386861 530436893 759789630 1040445072 1321100514 1601755956 1601755956 7756962475 43000123284 248369601964

32

1 16 135 798 3710 14443 48958 148376 409747 1045726 2492965 5597670 11915412 24170921 46923592 87438703 156764345 270808352 451415231 726366162 1129378998 1694171592 2420743944 3309096054 4197448164 5085800274 5085800274 24708004563 137322066880 794919136728

33

1 16 136 815 3857 15323 53079 164502 464648 1212835 2956976 6790112 14782443 30671003 60907973 116168626 213318904 377842627 646268702 1068598332 1708305016 2642334549 3947632726 5624199547 7672035012 9719870477 11767705942 11767705942 57390010121 319962207200 1857112329035

34

1 17 152 952 4690 19341 69391 222313 648024 1742876 4372173 10318287 23065169 49104416 100002420 195522490 368095838 668641429 1173734695 1992776249 3275160599 5210276148 8027505194 11956230035 16996450671 23148167102 29299883533 35451599964 35451599964 173405214133 969115652680 5636545892795

35

1 17 153 968 4828 20193 73584 239724 711191 1947972 4978637 11973494 27281073 59212619 122974951 245267721 471176784 873995390 1567973198 2723993112 4585248117 7482918115 11836169925 18152157283 26938033925 38193799851 51919455061 65645110271 79370765481 79370765481 389606249120 2183762223357 12732900345928

36

1 18 170 1122 5812 25160 94649 317690 969536 2728323 7156370 17646598 41189915 91523745 194489554 396758245 779413313 1478212670 2712350050 4821715994 8312658415 13903743163 22571514590 35550495550 54333187399 80412091493 113787207832 154458536416 195129865000 235801193584 235801193584 1160606285961 6519793249890 38088111350198

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1 19 189 1309 7089 31972 124916 434297 1369777 3976147 10740847 27238172 65304229 148880694 324305509 677664857 1362853058 2645019334 4965112696 9031236206 15936810614 27305547370 45436421636 73448645298 115281666966 175695171974 259448845656 366542688012 496976699042 627410710072 757844721102 757844721102 3738436950162 21039916860397 123110229387834

38

1 19 190 1329 7293 33405 132744 469812 1509257 4463741 12288337 31761912 77619334 180370197 400455893 852853078 1748127880 3458281581 6618064889 12274774300
22099544050 38660133657 65755211053 108748635716 174903741802 273383338145 415249708257 611565135650 862329620324 1167543162279 1472756704234 1777970246189
1777970246189 8794596642737 49608052563411 290838337577435

39

1 20 209 1520 8643 40919 167739 611416 2019974 6136289 17332058 45920332 114935340 273350598 620724418 1351295518 2829880096 5717545469 11171954192 21154191386
38880806818 69459803276 120710571239 204167769842 336085207841 538439724578 838931189981 1270982504564 1868016568841 2630033382812 3557032946477 4484032510142
5411032073807 5411032073807 26822143250162 151564955142510 889949312454085

40

1 20 210 1539 8835 42291 175491 648159 2171486 6693444 19192670 51641697 131311027 317350814 732471734 1621044895 3451655264 7091676941 14093791692 27149544464
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10163469325299 12258400514566 12258400514566 60920541130023 344986952773120 2029460152095008

41

1 21 230 1750 10393 51319 219177 831657 2858295 9027414 26495384 72908803 189454934 467628350 1101746225 2487837504 5402841184 11317817491 22926844195 45009438666
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42

1 21 231 1770 10605 52896 228399 876762 3050072 9754940 29003890 80878724 213037073 533164323 1273982915 2918317607 6430780137 13671944857 28114166872 56039174568
108476249149 204236184925 374492605215 669458589851 1167706570269 1988131492187 3304557245870 5360343096395 8482990113916 13084141173664 19659580955947
28705094141073 40220680729042 54206340719854 68192000710666 82177660701478 82177660701478 410192870407642 2331450441142669 13759389839553008

43

1 22 252 2002 12395 63714 282891 1114578 3973383 13005357 39529301 112578804 302616830 772366978 1881080241 4389921117 9851553907 21323381072 44630073183
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1 23 275 2275 14630 77919 357637 1453576 5335390 17950070 55994294 163449322 449793848 1174065730 2921696190 6961555187 15939972308 35182541529 75055125077
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82821270404669 127137364497861 190287429259482 276975051974561 387200232643098 520962971265093 654725709887088 788488448509083 788488448509083 3949123756172273
22510248841614181 133180667145777072

45

1 23 276 2299 14924 80388 373672 1539569 5732129 19569316 61959853 183592367 512869277 1358956428 3432856444 8302674426 19296394899 43230012635 93607286054
196359711535 399835023653 791676555891 1526541094553 2870279425425 5268251608196 9447496849116 16564525677833 28410502134351 47673815966109 78260044835181
125611916525476 197029271149938 301669061150546 450545351298314 654672206364256 914049626348372 1228677611250662 1543305596152952 1857933581055242 1857933581055242
9323007145147823 53226774117504832 315356241137505268

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1 24 299 2576 17248 95627 456764 1931126 7369154 25757578 83415662 252589666 720510694 1948020344 5017766580 12367492316 29276512550 66774021477 147142964276
314012249820 650312464908 1309305366167 2566714654818 4905914264341 9153170611710 16686086979300 29744549910999 51879629222489 88575883225660 148047069167157
242205258881060 387629954441564 606537203815659 926779600863130 1381846285336557 2005226396988520 2796919935819019 3756926901828054 4716933867837089
5676940833846124 5676940833846124 28528983340544231 163083012688976577 967303800643232882

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1 24 300 2599 17526 97977 472339 2017158 7780466 27503739 90121123 276209422 797669921 2183854663 5697112131 14223020056 34106482135 78807684049 175945881979
380455016732 798436401594 1629191402025 3237270277121 6272689340126 11866254471919 21938184787327 39671955909325 70219007961394 121713689043518 206677804957301
343814775180204 560193395088902 893431598182641 1393880218306595 2124622989651866 3161476546755484 4580257524154479 6380965921848851 8563601739838600
10746237557828349 12928873375818098 12928873375818098 65090946123968157 372660029112107029 2213388970068123188

48

1 25 324 2900 20148 115782 572693 2505429 9886833 35717162 119492057 373615962 1099950907 3068024400 8149385982 20704870690 50504182741 118656012107 269261034031
591609507478 1261236130936 2613735898730 5273912472439 10375809562966 19927958510660 37404254154983 68674246201554 123427304622986 217285788958516 374843022706012
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1 26 350 3248 23350 138575 706560 3180881 12896843 47802986 163879154 524479299 1578903249 4499078134 12198654979 31612185321 78597230908 188104425814 434580603666
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1241422721939015 2091990056378083 3457050318115498 5600512834181474 8888357244133662 13809673416917547 20976661450866448 31124631673701518 44988894413143910
62569449669193624 83866297441850660 105163145214507696 126459992987164732 126459992987164732 638328433134211905 3662696498655496841 21797112395398269352

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1 26 351 3275 23722 142080 732021 3333114 13677488 51334286 178260247 578018328 1763283700 5092007315 13992692937 36751640715 92610285178 224630909233 525948799055
1191685614169 2618623516774 5591300193966 11620279983913 23541349741698 46550543186632 89948377069182 170008548865771 314582661137100 570303302170316
1013553026067305 1766671988983344 3021182211762301 5069806439214572 8347281596037755 13479966839380061 21335451611403725 33072163691848417 50139369198031242
74277172584846740 107225678307189451 148984886365059375 199554796758456512 250124707151853649 300694617545250786 300694617545250786 1520038781147258614
8732865467429920500 52028134169251235063

51

1 27 377 3627 27025 166198 877910 4093287 17181483 65900076 233664252 773055109 2404502421 7075492460 19801025294 52936617079 135713727441 334755157417
796742372147 1834388939703 4094600892873 8878283817176 18732507236347 38519194065134 77296420337248 151550764933044 290619640496914 545574588048956
1003433551859760 1809332167971123 3200211099309170 5554503449900023 9463233290261825 15827936326046926 25985402709934231 41859241996771710 66111448098718908
102293964240389455 154848682958851066 229107446103625541 330402095524234680 458732631220678483 614099053192956950 769465475165235417 924831897137513884
924831897137513884 46805663362844404062 26917478370881040545 160509643506854706934

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1 27 378 3653 27378 169530 902448 4243290 17972522 69593883 249228496 833084822 2618815484 7790111775 22043007584 59594937635 154528205303 385559536948
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1 28 405 4032 31057 197262 1075340 5170720 22370235 88391047 322725213 1098988695 3517129748 10645125036 30631909701 84179378889 221776301917 562011966572
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2242067540520303 4149360569806271 7534760973283026 13432122459536802 23517426040105045 40451458093423691 68368691635649021 113549109243521491 185272967629271537
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5347484926360009719 6424145833371802181 6424145833371802181 32595600837982736893 187864998899925074787 1122428422670255691408

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1 29 434 4464 35469 232024 1300689 6422345 28493835 115318145 430774429 1499341986 4899924065 15131590755 44392852225 124294526146 333424816936 859841392869
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4052703374898527 7604818592065590 14001316273705791 25306861189509203 44927000833597386 78368397797972379 134354364456677557 226414616066539229 375068158390357853
610599309842548868 976426858157233143 1532473217056327426 2358533582917634793 3554275934774844648 5239241034317532723 7532969643235274750 10435461761528070729
13946717389195920660 17457973016863770591 20969228644531620522 20969228644531620522 106497472841098178757 614301773874020943510 3672921591387837707209

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1 29 435 4494 35928 236817 1339182 6676137 29925170 122420567 462422575 1627949522 5382285108 16817436648 49926079442 141459536739 384025095427 1002233876025
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5383928667533573 10224435345624382 19052619072410766 34858594087242498 62651563999173240 110664289615309721 192167570957348837 328128290167781870
550992390312512679 909874165086636716 1477151859423380867 2356469581006204118 3690739211780898221 5670142319467688360 8532130157561233151 12561423665330624642
18042743782044954881 24976090507704223868 33361463842308431603 41746837176912639338 50132210511516847073 50132210511516847073 254909121683580994882
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1 30 464 4930 40453 273231 1581174 8057569 36883899 153960240 592950844 2126910820 7160342589 22768738611 68753124180 198050065342 546372370944 1448466522793
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9073570646885569 17455260230279908 32947987579952722 61061267030433395 111167824241979852 198918676989065069 349964250008995635 605550897012672015
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38885152391956684668 55808794440336702974 77208878493855568702 103085404552513281852 128961930611170995002 154838456669828708152 154838456669828708152
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1 30 465 4959 40890 277788 1618101 8305119 38311364 161230719 626291680 2266579069 7700908370 24719391358 75365035617 219232778782 610850717206 1635776426737
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